Electric Vehicle Routing: Subpath-Based Decomposition

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15 October 2023

Motivation, problem setting

Biden administration plan seeks elimination of transportation emissions

calls for a transition to electric vehicles and more walkable neighborhoods by 2050

A 40-ton Mercedes-Benz e-truck just drove 1,000 km with only one stop to charge

Michelle Lewis | Oct 5 2023 - 10:48 am PT | 66 Comments

LOGISTICS REPORT

California's Electric-Truck Drive Draws Startups Building Charging Networks

n aggressive emissions-slashing mandate means thousands of arging sites are needed in the coming years

Paul Berger Follow

y 29, 2023 7:00 am ET

Biden administration plan calls for $5 billion network of electric-vehicle chargers along interstates

Grants included in the infrastructure law will help states build a charging network designed to reach highways in almost every corner of the country

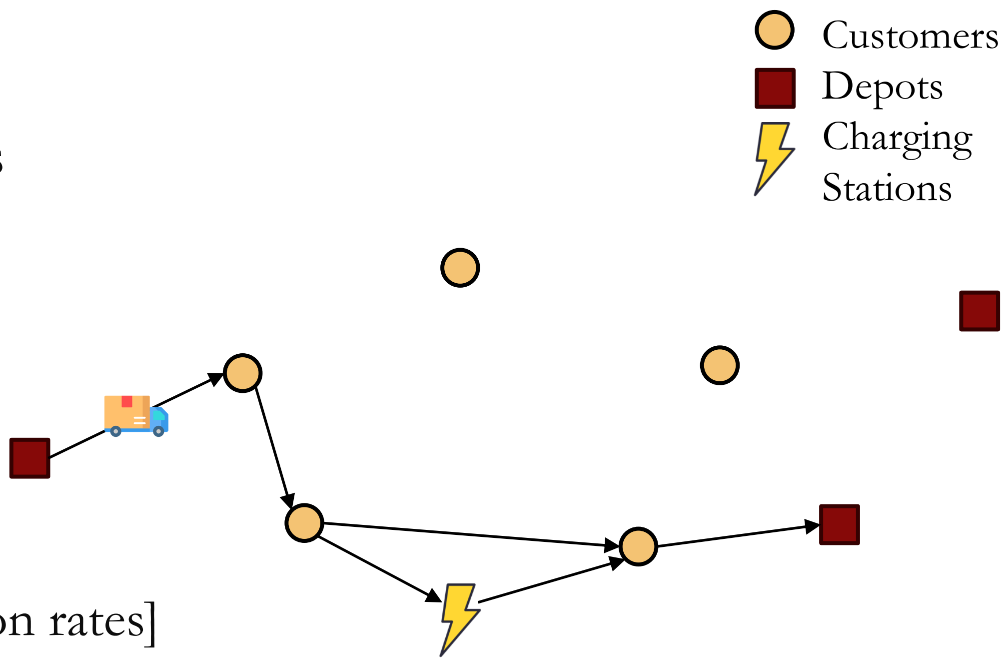
By Ian Duncan

Updated February 10, 2022 at 1:46 p.m. EST | Published February 10, 2022 at 5:00 a.m. EST

New routing algorithms for electrified logistics

Problem description

- Vehicle routing problem with electric vehicles, in continuous time and charge
  - Multiple depots
  - Multiple customers
  - Multiple charging stations



- <u>Assumptions:</u>
  - No time windows
  - Linear charging dynamics
    [Possibly non-linear depletion rates]

Contributions

Electric vehicle routing: subpath-based decomposition algorithm

Modeling	Electric vehicle routing: Semi-infinite set-partitioning formulation with continuous time and continuous charge
Optimization	• Subpath-based decomposition algorithm for column generation subproblem • Acceleration strategy via adaptive route relaxations to obtain elementary paths • Cutting planes to strengthen linear relaxation
Computational results	Significantly outperforms path-based benchmark, and scales to realistic problem instances
Practical impact	Benefits over "business-as-usual" routing operations

Semi-finite set-partitioning model

$$
\begin{aligned}
\min \quad & \sum_{p\in\mathcal{P}} c^p z^p & & \text{(minimize total cost of paths)} \\
\text{such that} \quad & \sum_{p\in\mathcal{P}} \alpha_j^p z^p = v_j^{\text{start}} & \forall\, j \in \mathcal{V}_D \quad & \text{(each depot } j \text{ starts with } v_j^{\text{start}} \text{ vehicles)} \\
& \sum_{p\in\mathcal{P}} \beta_j^p z^p \geq v_j^{\text{end}} & \forall\, j \in \mathcal{V}_D \quad & \text{(each depot } j \text{ ends with at least } v_j^{\text{start}} \text{ vehicles)} \\
& \sum_{p\in\mathcal{P}} \gamma_i^p z^p = 1 & \forall\, i \in \mathcal{V}_C \quad & \text{(each customer served once)} \\
& z^p \in \{0,1\} & \forall\, p \in \mathcal{P} \quad &
\end{aligned}
$$

- Set-partitioning formulation with path-based variables z^p
- Infinitely many variables
  - **Discrete** routing and timing decisions (as in traditional VRP)
  - **Continuous** charging decisions (new to E-VRP)

Column generation

Restricted Master Problem

$$\min \sum_{p\in\mathcal{P}'} c^p z^p$$

$$\text{such that} \quad \sum_{p\in\mathcal{P}'} \alpha_j^p z^p = v_j^{\text{start}} \quad \forall\, j \in \mathcal{V}_D \quad [\boldsymbol{\kappa}]$$

$$\sum_{p\in\mathcal{P}'} \beta_j^p z^p \geq v_j^{\text{end}} \quad \forall\, j \in \mathcal{V}_D \quad [\boldsymbol{\mu}]$$

$$\sum_{p\in\mathcal{P}'} \gamma_i^p z^p = 1 \quad \forall\, i \in \mathcal{V}_C \quad [\boldsymbol{\nu}]$$

$$z^p \in \{0,1\} \quad \forall\, p \in \mathcal{P}'$$

dual values $\boldsymbol{\kappa}, \boldsymbol{\mu}, \boldsymbol{\nu}$

Subproblem

$$\min_{p\in\mathcal{P}} \left\{ \bar{c}^p := c^p - \kappa_{\text{start}(p)} - \mu_{\text{end}(p)} - \sum_{i\in\mathcal{V}_C} \gamma_i^p \nu_i \right\}$$

paths not in $\mathcal{P}'$

Traditionally: solves an E-RCSPP by dynamic programming

- **Q:** How to guarantee finite termination?

Subpath-based decomposition in the pricing problem

from depot/charging station to depot/charging station

Pricing problem variables \ Master problem variables	Path	Subpath
Path	Column generation e.g. [1] for EVRPTW	CG for extended formulations [4]
Subpath	**This work** **Q:** How can you build paths from subpaths?	CG *on* extended formulations, e.g. [2] for PDP, [3] for ride-sharing

[1] Desaulniers, G., Errico, F., Irnich, S., & Schneider, M. (2016). Exact Algorithms for Electric Vehicle-Routing Problems with Time Windows. *Operations Research*, *64*(6), 1388–1405. https://doi.org/10.1287/opre.2016.1535

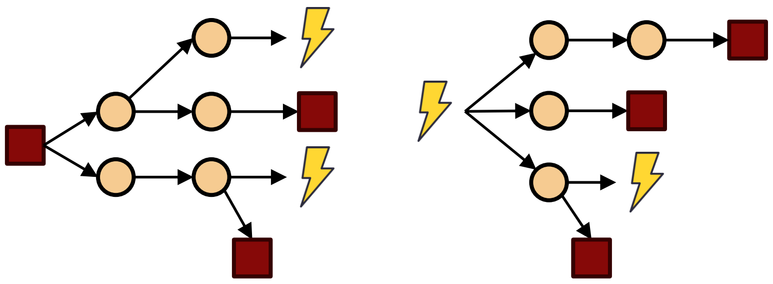
[2] Alyasiry, A. M., Forbes, M., & Bulmer, M. (2019). An Exact Algorithm for the Pickup and Delivery Problem with Time Windows and Last-in-First-out Loading. *Transportation Science*, *53*(6), 1695–1705. https://doi.org/10.1287/trsc.2019.0905

[3] Zhang, W., Jacquillat, A., Wang, K., & Wang, S. (2022). Routing Optimization with Vehicle-Customer Coordination. *SSRN Electronic Journal*. https://doi.org/10.2139/ssrn.4208397

[4] Sadykov, R., & Vanderbeck, F. (2013). Column generation for extended formulations. *EURO Journal on Computational Optimization*, *1*(1), 81–115. https://doi.org/10.1007/s13675-013-0009-9

Key idea: generate-and-stitch

Step 1: Generate **subpaths**
- Label-setting, with charge and time taken as domination criteria

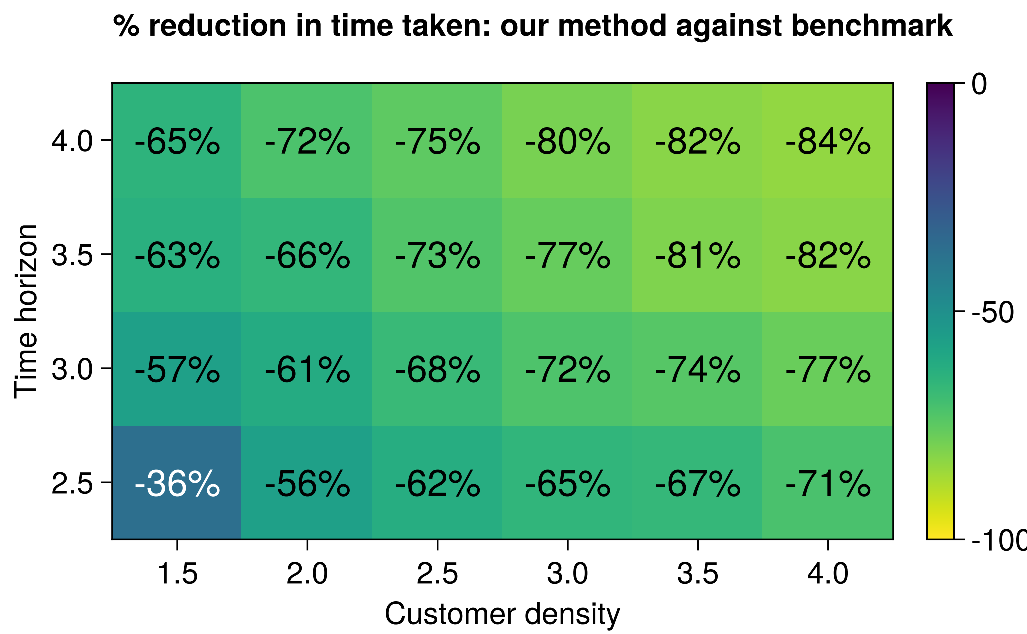


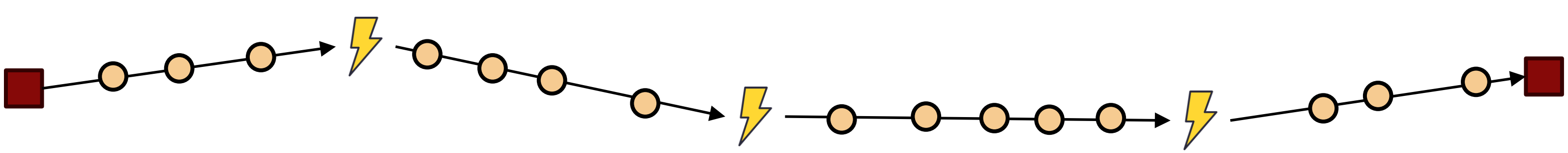
Step 2: Stitch subpaths into **paths**
- A subpath valid at time 0 is still valid at time t with the same reduced cost
- Charging action between subpaths is the minimum possible
- Reduced cost of path = r.c. of subpaths + r.c. of charging actions

Theorem: with this, CG finitely converges to LP optimum of EVRP

Comparison to benchmark

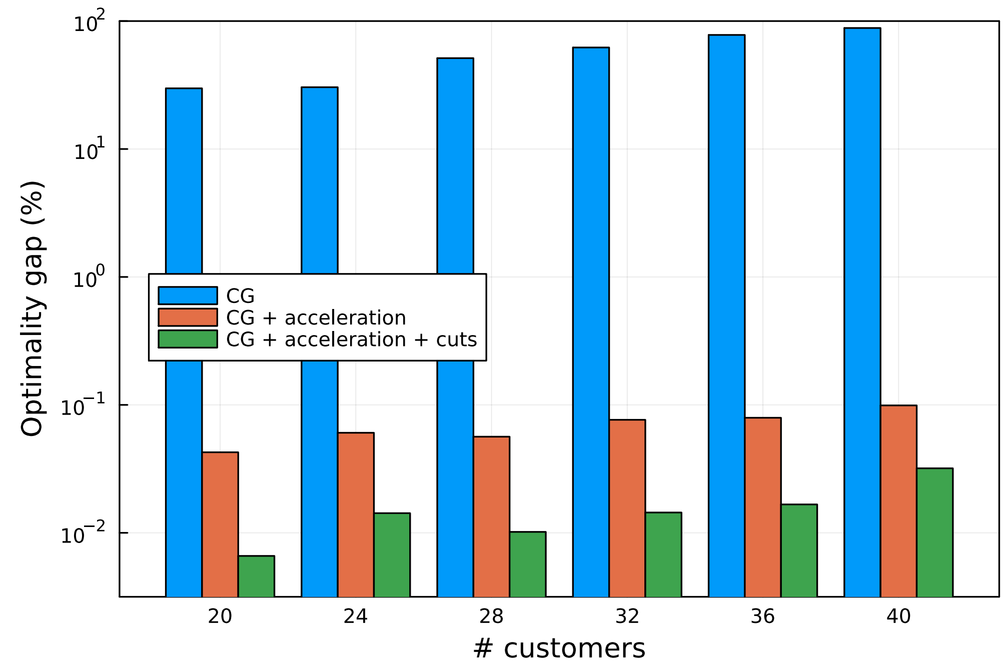
- Significant speedups against path-based benchmark
- Stronger improvement with:
  - Higher customer density ≈ **longer subpaths**
  - Longer time horizon ≈ **more subpaths per path**

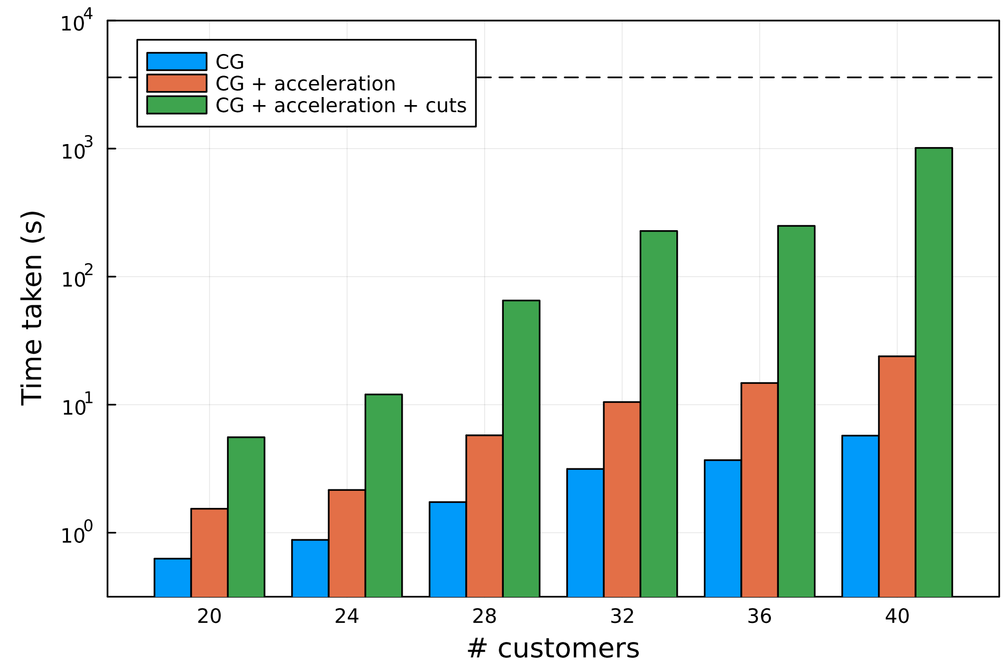




Computational results

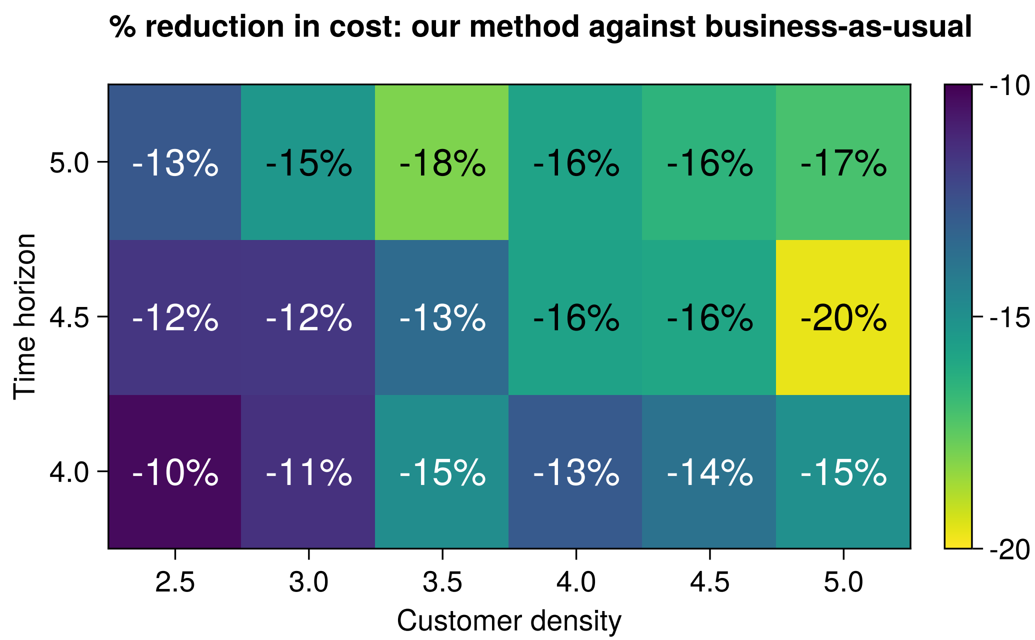
- Additional algorithmic elements
  - Adaptive ng-route: Acceleration strategy for finding the LP relaxation of elementary paths
  - Cutting-plane algorithm to tighten LP relaxation
- Small optimality gaps in manageable runtimes





The benefits of optimization

- Improvement compared to business-as-usual solution:
  - Solve a VRP w/o charge
  - Then optimize charging stations with fixed routes
- Benefit of **jointly** optimizing charging and routing decisions



Contributions

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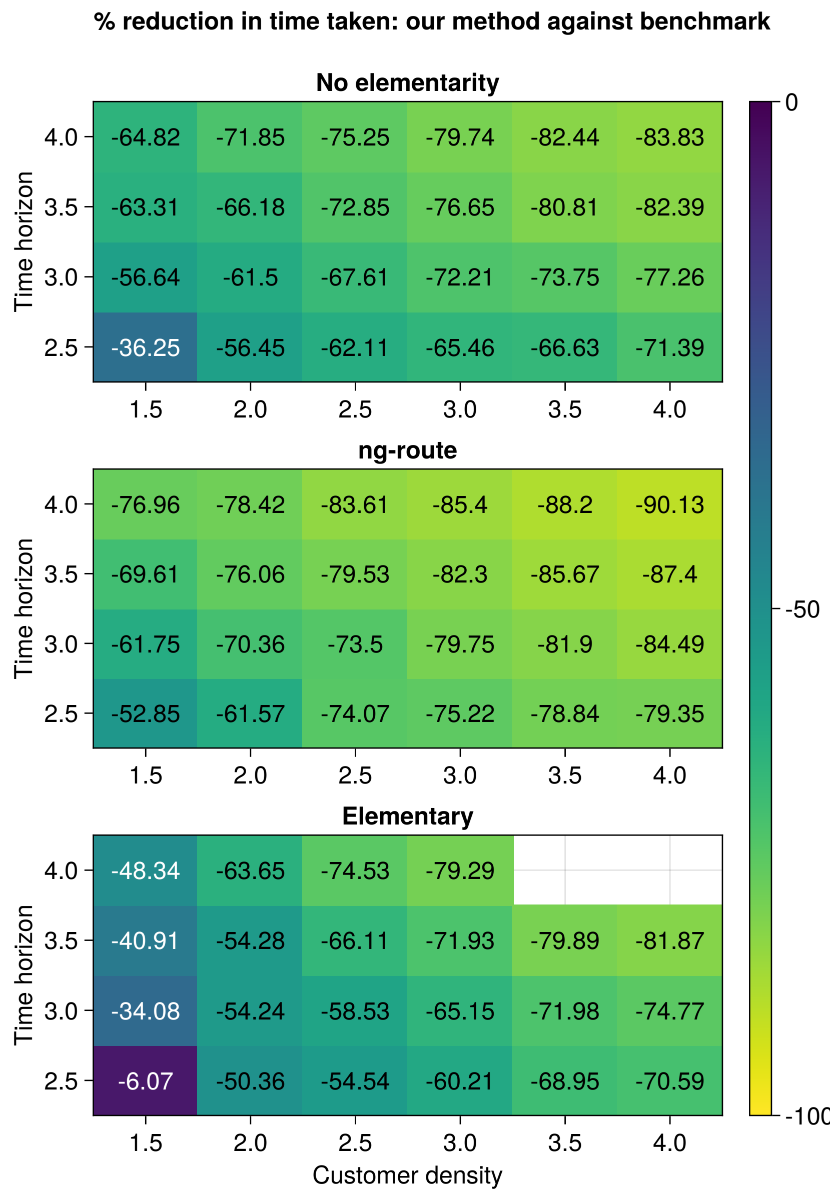
Practical impact	Benefits over separately optimizing routing and charging

Additional slides

Comparison to benchmark

- Benchmark time taken against Desaulniers (2016)[1]: generating **paths**
- Improvement over benchmark in all settings and relaxations
- Bigger improvement with:
  - Greater customer density ≈ **longer subpaths**
  - Greater time horizon ≈ **more subpaths per path**



Intuition: solving 1 DP with large* state space > solving m DPs with smaller* state space

[1] Desaulniers, G., Errico, F., Irnich, S., & Schneider, M. (2016). Exact Algorithms for Electric Vehicle-Routing Problems with Time Windows. *Operations Research*, *64*(6), 1388–1405. https://doi.org/10.1287/opre.2016.1535

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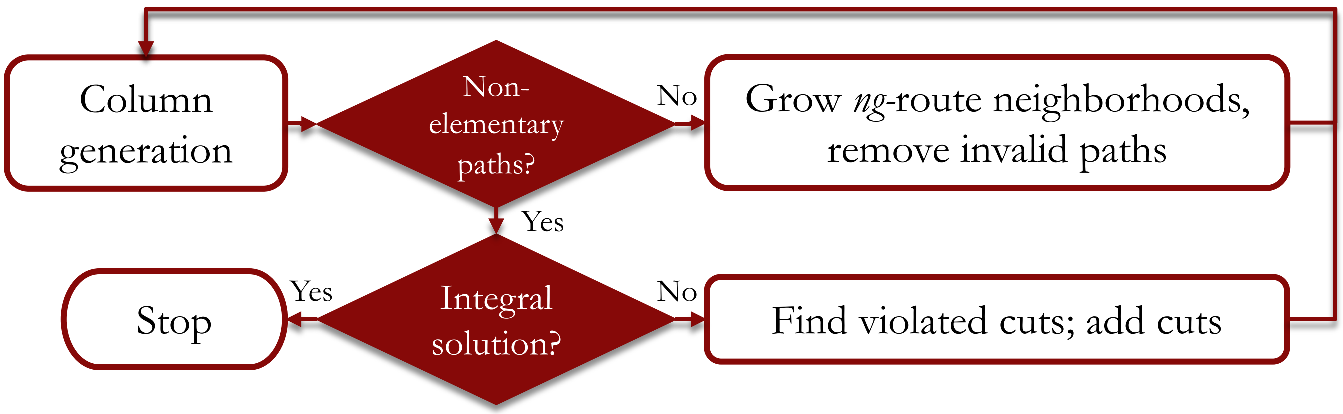
Summary

Electric vehicle routing: subpath-based decomposition algorithm

Modeling

Electric vehicle routing: semi-infinite set-partitioning formulation with continuous time and continuous charge

Optimization



Computational results

Significantly outperforms path-based benchmark, and scales to realistic problem instances

Practical impact

Benefits over separately optimizing routing and charging

The elementarity constraint

- The E-RCSPP is very expensive!
  - One binary label for each customer, denoting if customer i visited
- Relaxations of the subproblem → looser definition of paths
  - No elementarity
  - *ng*-route relaxation[1]:
    - Each customer i has a neighborhood of nodes $i \in N_i \subset \mathcal{N}_C$
    - Between two visits to customer i, must visit customer j with $i \notin N_j$
    - Each partial path visiting nodes $i_0, i_1, \dots, i_k$ has an associated *ng*-set

$$\Pi(P) = \left\{ i_r \colon i_r \in \bigcap_{s=r+1}^{k} N_{i_s},\ r = 1, \dots, k-1 \right\} \bigcup \{i_k\}.$$

    - Set inclusion of *ng*-sets is a domination criterion

[1] Baldacci, R., Mingozzi, A., & Roberti, R. (2011). New Route Relaxation and Pricing Strategies for the Vehicle Routing Problem. *Operations Research*, *59*(5), 1269–1283.

ng-routes in generate-and-stitch

- Traditionally: Forward labelling keeps track of **forward** *ng*-sets

$$\Pi(P) = \left\{ i_r \colon i_r \in \bigcap_{s=r+1}^{k} N_{i_s},\ r = 1, \ldots, k-1 \right\} \bigcup \{i_k\}.$$

- Backward labelling (in bi-directional label-setting[1]) tracks **backward** *ng*-sets

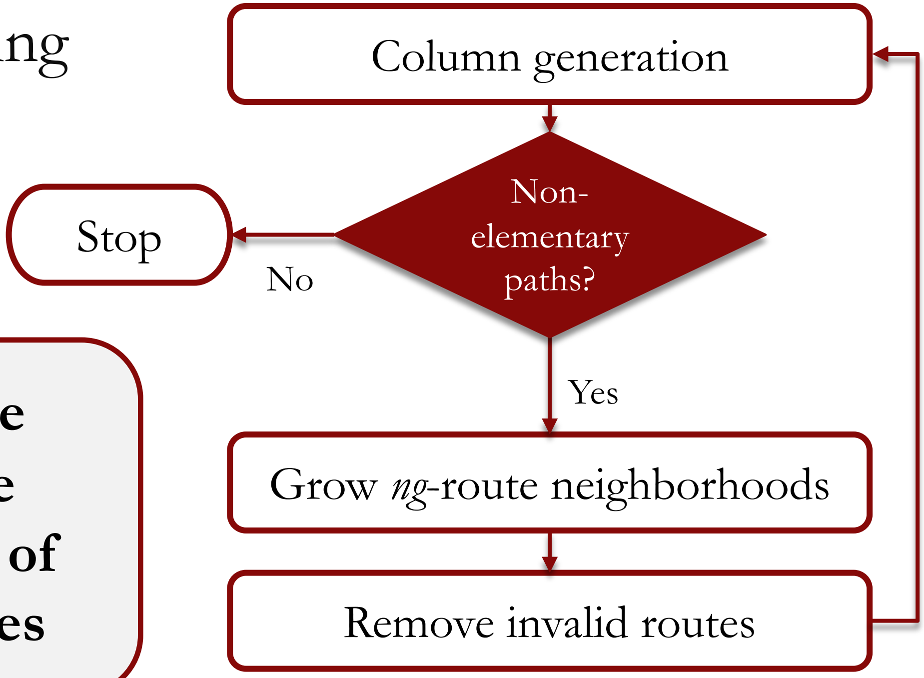
$$\Pi^{-1}(\bar{P}) = \{i_k\} \cup \left\{ i_r \colon i_r \in \bigcap_{s=k}^{r-1} N_{i_s},\ r = k+1, \ldots, h \right\}.$$

- Our method requires tracking both forward and backward *ng*-sets, since subpaths can be extended from the front and back
  - Larger state space in the "generate" step

[1] Baldacci, R., Mingozzi, A., & Roberti, R. (2011). New Route Relaxation and Pricing Strategies for the Vehicle Routing Problem. *Operations Research*, *59*(5), 1269–1283.

Acceleration: adaptive *ng*-routes

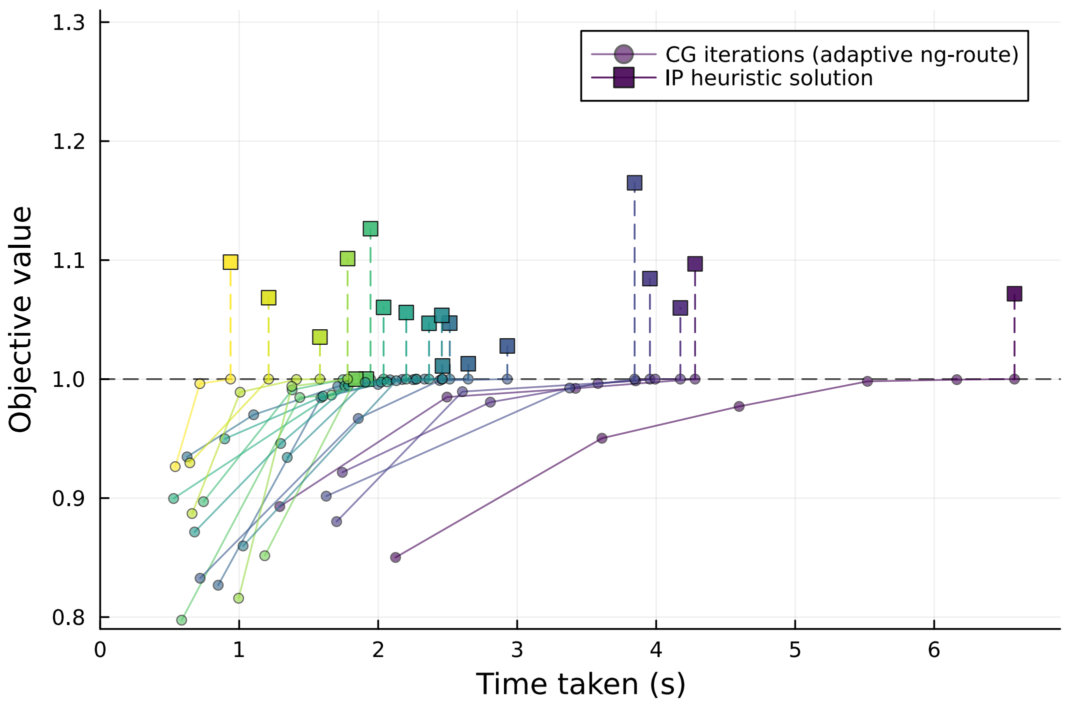
- Large *ng*-route neighborhoods → tight (and slow) relaxations
- **Idea:** start with a loose relaxation, tighten when necessary!
- Analog: cutting planes yielding nested relaxations



Proposition: This procedure terminates finitely, with the solution to the LP relaxation of EVRP with elementary routes

Computational results

- Adaptive *ng*-route procedure reaches the objective of the elementary relaxation (in much less time)!
- Sometimes: no integrality gap 😊
- Else: some integrality gap remains
  - Time for cuts!



Subset-row cuts

- At most $\lfloor n/k \rfloor$ routes visiting at least k out of n customers[1]
- *Non-robust* cuts which change subproblem structure
- Limited-memory subset-row cuts[2] include a *memory neighborhood* for each cut
  - Smaller state space
  - Weaker cuts

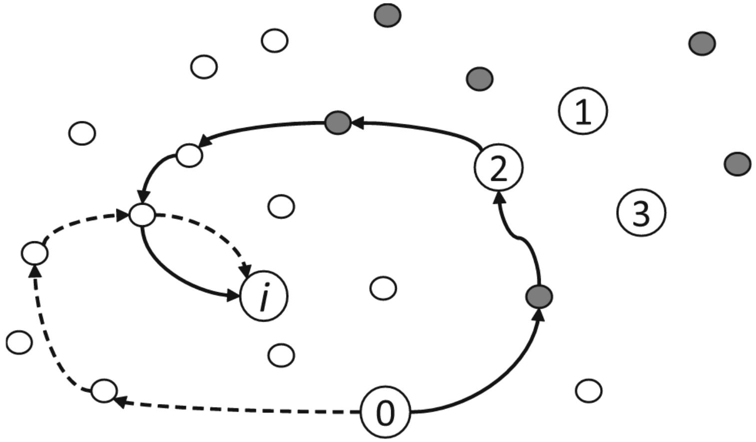


Fig. 2 Example illustrating the performance gain in the pricing when using lm-SRCs

[1] Jepsen, M., Petersen, B., Spoorendonk, S., & Pisinger, D. (2008). Subset-Row Inequalities Applied to the Vehicle-Routing Problem with Time Windows. *Operations Research*, *56*(2), 497–511. https://doi.org/10.1287/opre.1070.0449

[2] Pecin, D., Pessoa, A., Poggi, M., & Uchoa, E. (2017). Improved branch-cut-and-price for capacitated vehicle routing. *Mathematical Programming Computation*, *9*(1), 61–100. https://doi.org/10.1007/s12532-016-0108-8

Computational results

- Cuts further close the optimality gap at the cost of more time
- IP solution obtained typically optimal
- *lm*-SR3 cuts provides an intermediate approach