Electric Vehicle Routing: Subpath-Based Decomposition

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Background and motivation

Biden administration plan seeks elimination of transportation emissions

calls for a transition to electric vehicles and more walkable neighborhoods by 2050

A 40-ton Mercedes-Benz e-truck just drove 1,000 km with only one stop to charge

Michelle Lewis | Oct 5 2023 - 10:48 am PT | 66 Comments

LOGISTICS REPORT

California's Electric-Truck Drive Draws Startups Building Charging Networks

aggressive emissions-slashing mandate means thousands of arging sites are needed in the coming years

Paul Berger Follow

y 29, 2023 7:00 am ET

Biden administration plan calls for $5 billion network of electric-vehicle chargers along interstates

Grants included in the infrastructure law will help states build a charging network designed to reach highways in almost every corner of the country

By Ian Duncan

Updated February 10, 2022 at 1:46 p.m. EST | Published February 10, 2022 at 5:00 a.m. EST

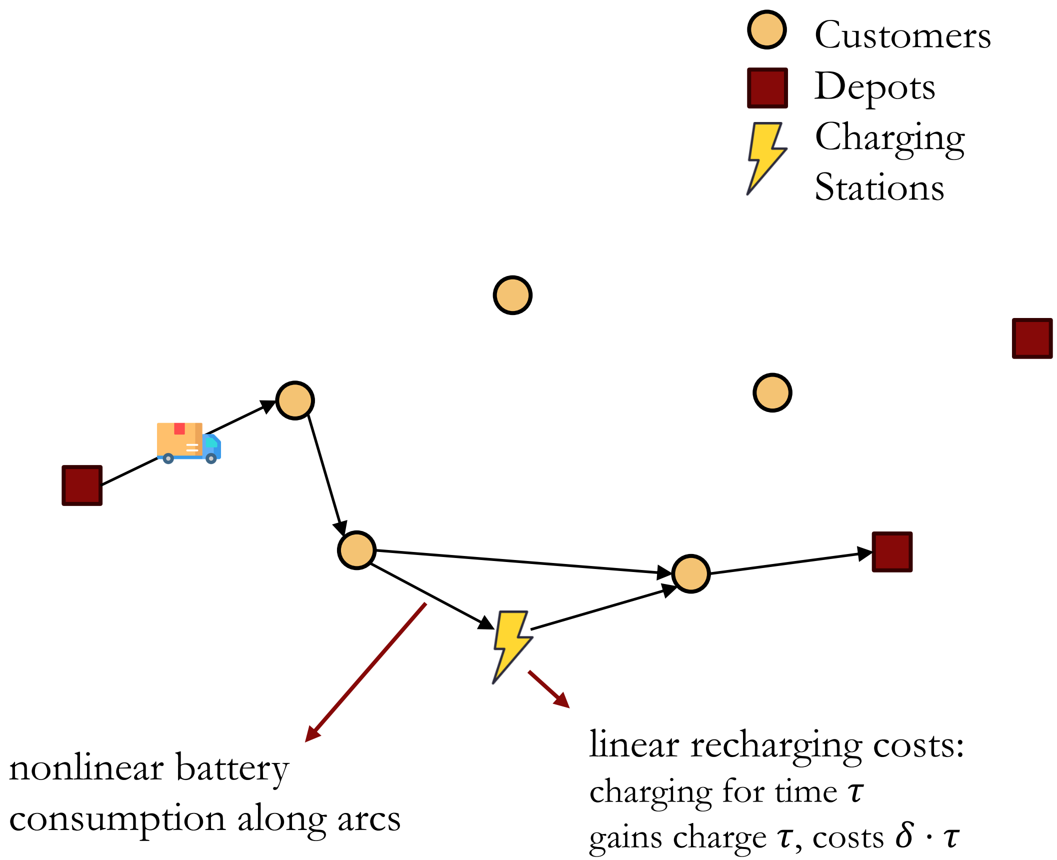
New routing algorithms for electrified logistics

Contributions

Electric vehicle routing: subpath-based column generation algorithm

Modeling	Semi-infinite formulation for electric vehicle routing: discrete routing decisions, continuous charging decisions
Optimization	• Subpath-based decomposition: two-level label-setting pricing algorithm for column generation subproblem • Forward and backward domination criteria enabling tighter relaxations: *ng*-relaxations & cutting planes • Guarantees of exactness and finite convergence
Computational results	Significantly outperforms path-based benchmark, and scales to realistic problem instances
Practical impact	Benefits of integrated routing and charging optimization

Electric Vehicle Routing Problem



Semi-infinite set-partitioning model

$$\begin{aligned}
\min \quad & \sum_{p\in\mathcal{P}} c^p z^p && && \text{(minimize total cost of paths)} \\
\text{s.t.} \quad & \sum_{p\in\mathcal{P}} \mathbb{1}\left(n^p_{\text{start}} = j\right) z^p = v_j^{\text{start}} && \forall \text{ depots } j && \text{(each depot } j \text{ starts with } v_j^{\text{start}} \text{ vehicles)} \\
& \sum_{p\in\mathcal{P}} \mathbb{1}\left(n^p_{\text{end}} = j\right) z^p \geq v_j^{\text{end}} && \forall \text{ depots } j && \text{(each depot } j \text{ ends with at least } v_j^{\text{end}} \text{ vehicles)} \\
& \sum_{p\in\mathcal{P}} \gamma_i^p z^p = 1 && \forall \text{ customers } j && \text{(each customer served once)} \\
& z^p \in \mathbb{Z}_+ && \forall\, p \in \mathcal{P} &&
\end{aligned}$$

- Set-partitioning formulation with path-based variables z^p
- Infinitely many variables
  - **Discrete** routing and timing decisions (as in traditional VRP)
  - **Continuous** charging decisions (new to E-VRP)

Column generation

Restricted Master Problem

$$\min \sum_{p \in \mathcal{P}'} c^p z^p$$

$$\text{s.t.} \sum_{p \in \mathcal{P}'} \mathbb{1}\left(n^p_{\text{start}} = j\right) z^p = v^{\text{start}}_j \quad \forall \text{ depots } j \quad [\boldsymbol{\kappa}]$$

$$\sum_{p \in \mathcal{P}'} \mathbb{1}\left(n^p_{\text{end}} = j\right) z^p \geq v^{\text{end}}_j \quad \forall \text{ depots } j \quad [\boldsymbol{\mu}]$$

$$\sum_{p \in \mathcal{P}'} \gamma^p_i z^p = 1 \quad \forall \text{ customers } j \quad [\boldsymbol{\nu}]$$

$$z^p \in \mathbb{Z}_+ \quad \forall\, p \in \mathcal{P}'$$

dual values $\boldsymbol{\kappa}, \boldsymbol{\mu}, \boldsymbol{\nu}$

paths not in $\mathcal{P}'$

Pricing Problem

$$\min_{p \in \mathcal{P}} \left\{ \bar{c}^p := c^p - \kappa_{\text{start}(p)} - \mu_{\text{end}(p)} - \sum_{i \in \mathcal{V}_C} \gamma^p_i \nu_i \right\}$$

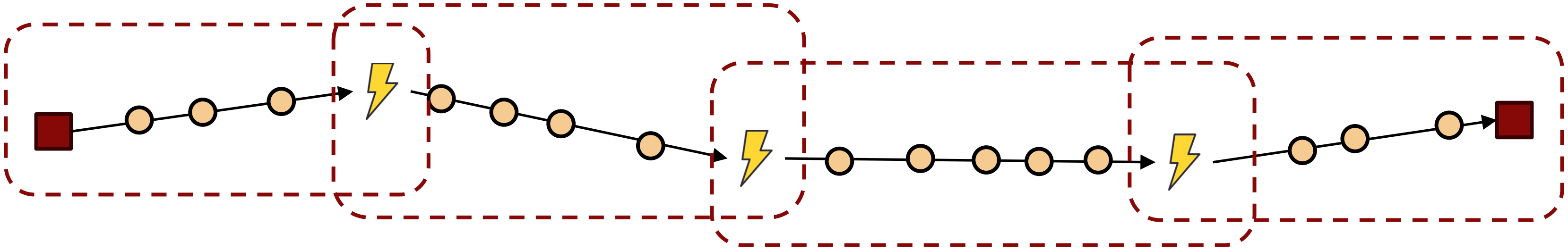
Algorithmic challenges

1. How to solve the pricing problem efficiently?
   - NP-hard Elementary Resource-Constrained Shortest Path structure

$$\min_{p \in \mathcal{P}} \left\{ \bar{c}^p := c^p - \kappa_{\text{start}(p)} - \mu_{\text{end}(p)} - \sum_{i \in \mathcal{V}_C} \gamma_i^p \nu_i \right\}$$

2. How to ensure finite convergence in column generation?
   - Infinitely many path-based variables: $\min \sum_{p \in \mathcal{P}} c^p z^p$
3. How to impose path elementarity?
   - Trade-off: relaxation strength vs. high-dimensional domination labels
4. How to eliminate fractional solutions?
   - Embedding limited-memory subset row inequalities $z^p \in \mathbb{Z}_+$

Pricing problem in CG



- Finding paths of negative reduced cost via DP
  - Resource-Constrained Shortest Path Problem (RCSPP)[1]

Extend partial paths along edges

Prune "dominated" paths using domination criteria

$$D(p) = \Big(\bar{c}(p),\ t(p),\ -b(p)\Big)$$

reduced cost | time | (negative of) charge

Challenges:

1. Grows exponentially with no. of customers
2. How to determine charging time? → Extra labels[2]

[1] Irnich, S., & Desaulniers, G. (2005). Shortest Path Problems with Resource Constraints. In G. Desaulniers, J. Desrosiers, & M. M. Solomon (Eds.), *Column Generation* (pp. 33–65). Springer US. https://doi.org/10.1007/0-387-25486-2_2
[2] Desaulniers, G., Errico, F., Irnich, S., & Schneider, M. (2016). Exact Algorithms for Electric Vehicle-Routing Problems with Time Windows. Operations Research, 64(6), 1388–1405. https://doi.org/10.1287/opre.2016.1535

Key idea: two-level label-setting

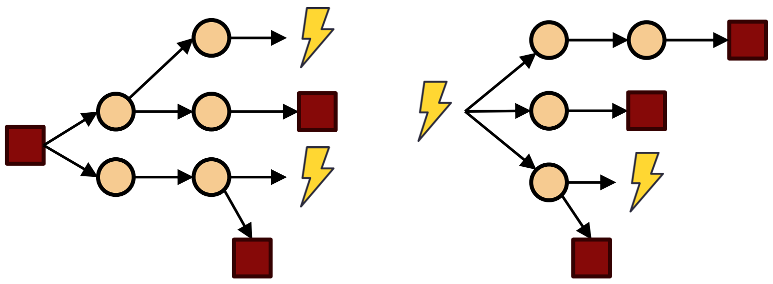
Level 1: Generate **subpaths** s

- Label-setting, with domination criteria:

$$D(s) = \left(\bar{c}(s),\ t(s),\ b(s)\right)$$

Level 2: Extend **paths** p along subpaths s

- A subpath valid at time 0 is still valid at time t with the same reduced cost
- WLOG, the charging decision between subpaths is the minimum possible
- Reduced cost of path = r.c. of subpaths + cost of charging



A closer look at domination

Our work:

Def: $s_1 \succcurlyeq_L s_2$ if: $L(s_1) \leq L(s_2)$

$$D(s) = \Big(\bar{c}(s),\ t(s),\ b(s)\Big)$$

reduced cost — time taken — charge taken

Traditionally[1]:

Def: $p_1 \succcurlyeq_L p_2$ if: $L(p_1) \leq L(p_2)$

$$D(p) = \Big(\bar{c}(p),\ t(p),\ -b(p)\Big)$$

reduced cost — time — (negative of) charge

$s_1 \succcurlyeq_L s_2$ implies:

2. $s_1 \oplus e \succcurlyeq_L s_2 \oplus e$ for any edge e
3. $p \oplus s_1 \succcurlyeq_L p \oplus s_2$ for any path p

$p_1 \succcurlyeq_L p_2$ implies:

1. $p_1 \oplus s \succcurlyeq_L p_2 \oplus s$ for any subpath s

Rigorous and generalizable framework for domination criteria

[1] Irnich, S., & Desaulniers, G. (2005). Shortest Path Problems with Resource Constraints. In G. Desaulniers, J. Desrosiers, & M. M. Solomon (Eds.), *Column Generation* (pp. 33–65). Springer US. https://doi.org/10.1007/0-387-25486-2_2

Key results

Theorem 1: Two-level label-setting finds negative reduced-cost paths, or certifies that none exists

Proof (sketch):

- Use properties 1 2 3 arising from domination criteria

Theorem 2: With two-level label-setting, CG converges **finitely** to LP optimum of EVRP

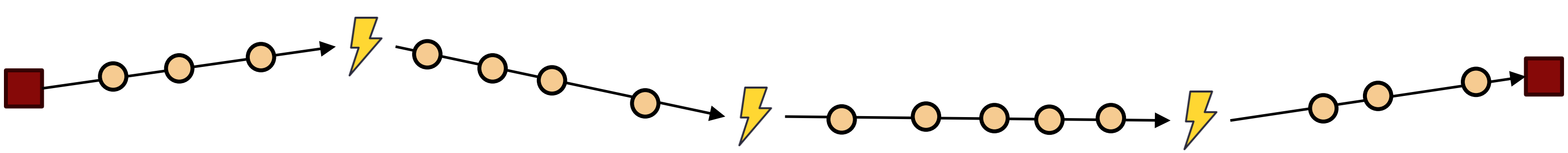
Proof (sketch):

- Infinitely many paths, but finitely many subpath sequences
- Once a path is added to RMP, no other path with the same subpath sequence will be added in future iterations

Comparison to benchmark

- Significant speedups against path-based benchmark
- Stronger improvement with:
  - Higher customer density ≈ **longer subpaths**
  - Longer time horizon ≈ **more subpaths per path**

% time reduction vs. path-based benchmark



Algorithmic challenges

1. How to solve the pricing problem efficiently?
   - NP-hard Elementary Resource-Constrained Shortest Path structure

$$\min_{p \in \mathcal{P}} \left\{ \bar{c}^p := c^p - \kappa_{\text{start}(p)} - \mu_{\text{end}(p)} - \sum_{i \in \mathcal{V}_C} \gamma_i^p \nu_i \right\}$$

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The elementarity constraint

- Ideally, each path serves each customer at most once! (elementarity)
- Affects the structure of the label-setting in the pricing problem:

Option 1:
Ignore elementarity

Option 2:
Enforce elementarity

One binary resource per customer[1]:

$$D(p) = \left(\bar{c}(p),\ t(p),\ -b(p),\ \overbrace{\gamma_1^p, \ldots, \gamma_n^p}\right)$$

Option 1:

- \+ Computationally cheap!
- − Good LP solutions, but bad IP solutions

Option 2:

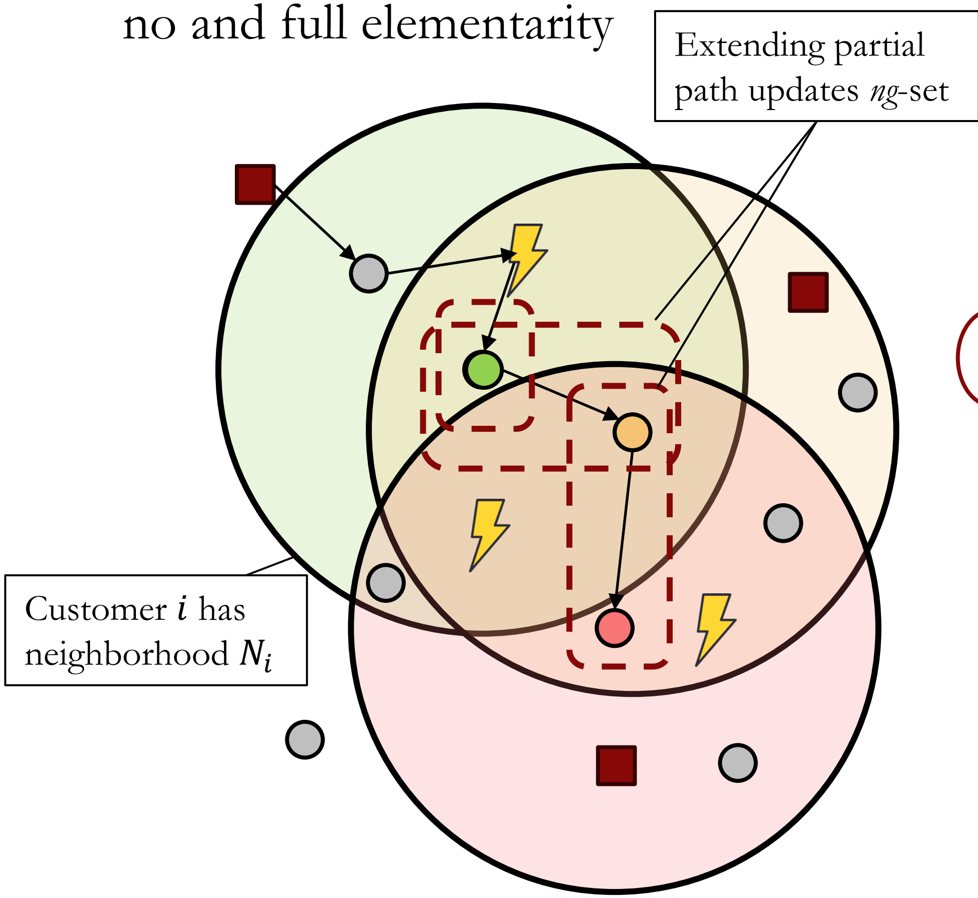
- − Expensive: NP-hard[2]
- \+ Better IP solutions

[1] Beasley, J. E., & Christofides, N. (1989). An algorithm for the resource constrained shortest path problem. *Networks*, *19*(4), 379–394. https://doi.org/10.1002/net.3230190402
[2] Dror, M. (1994). Note on the Complexity of the Shortest Path Models for Column Generation in VRPTW. *Operations Research*, *42*(5), 977–978. https://doi.org/10.1287/opre.42.5.977

Adaptive[2] *ng*-relaxations[1]

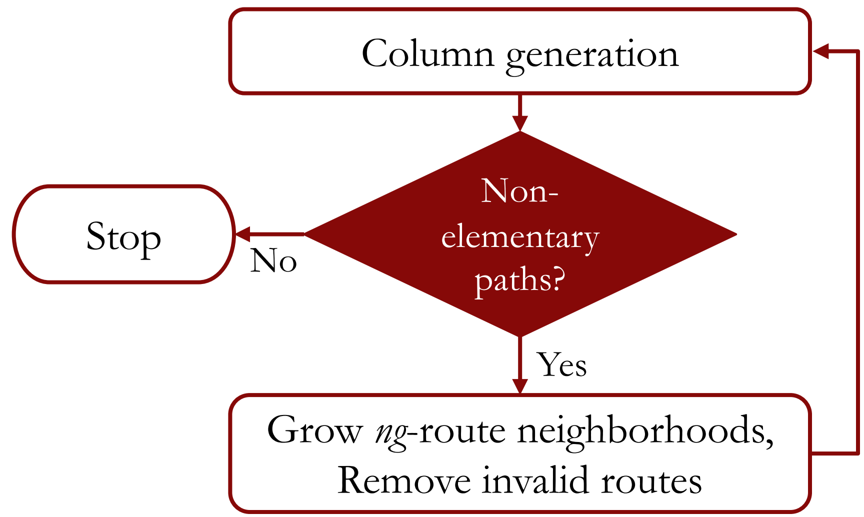
***ng*-route relaxation[1]:**

Interpolates between no and full elementarity



Nested *ng*-route relaxations

Start with a loose *ng*-relaxation, tighten when necessary[2]!



Prop: This yields the solution to the LP relaxation of EVRP with elementary routes

[1] Baldacci, R., Mingozzi, A., & Roberti, R. (2011). New Route Relaxation and Pricing Strategies for the Vehicle Routing Problem. *Operations Research*, *59*(5), 1269–1283.

[2] Martinelli, R., Pecin, D., & Poggi, M. (2014). Efficient elementary and restricted non-elementary route pricing. *European Journal of Operational Research*, *239*(1), 102–111. https://doi.org/10.1016/j.ejor.2014.05.005

ng-routes in two-level label-setting

Traditionally:

- **Forward** *ng*-sets in label-setting algorithms [1]

$$\Pi(P) = \left\{ i_r \colon i_r \in \bigcap_{s=r+1}^{k} N_{i_s},\ r = 1, \ldots, k-1 \right\} \bigcup \{i_k\}.$$

- **Backward** *ng*-sets in bidirectional label-setting algorithms [1]

$$\Pi^{-1}(\bar{P}) = \{i_k\} \cup \left\{ i_r \colon i_r \in \bigcap_{s=k}^{r-1} N_{i_s},\ r = k+1, \ldots, h \right\}.$$

Our work:

- Path domination criteria include forward *ng*-set inclusions:

$$D(p) = \left(\bar{c}(p),\ t(p),\ -b(p),\ \{\mathbb{1}\,(i \in \Pi(p))\}_i \right)$$

- Subpath domination criteria include **both** forward and backward *ng*-sets:

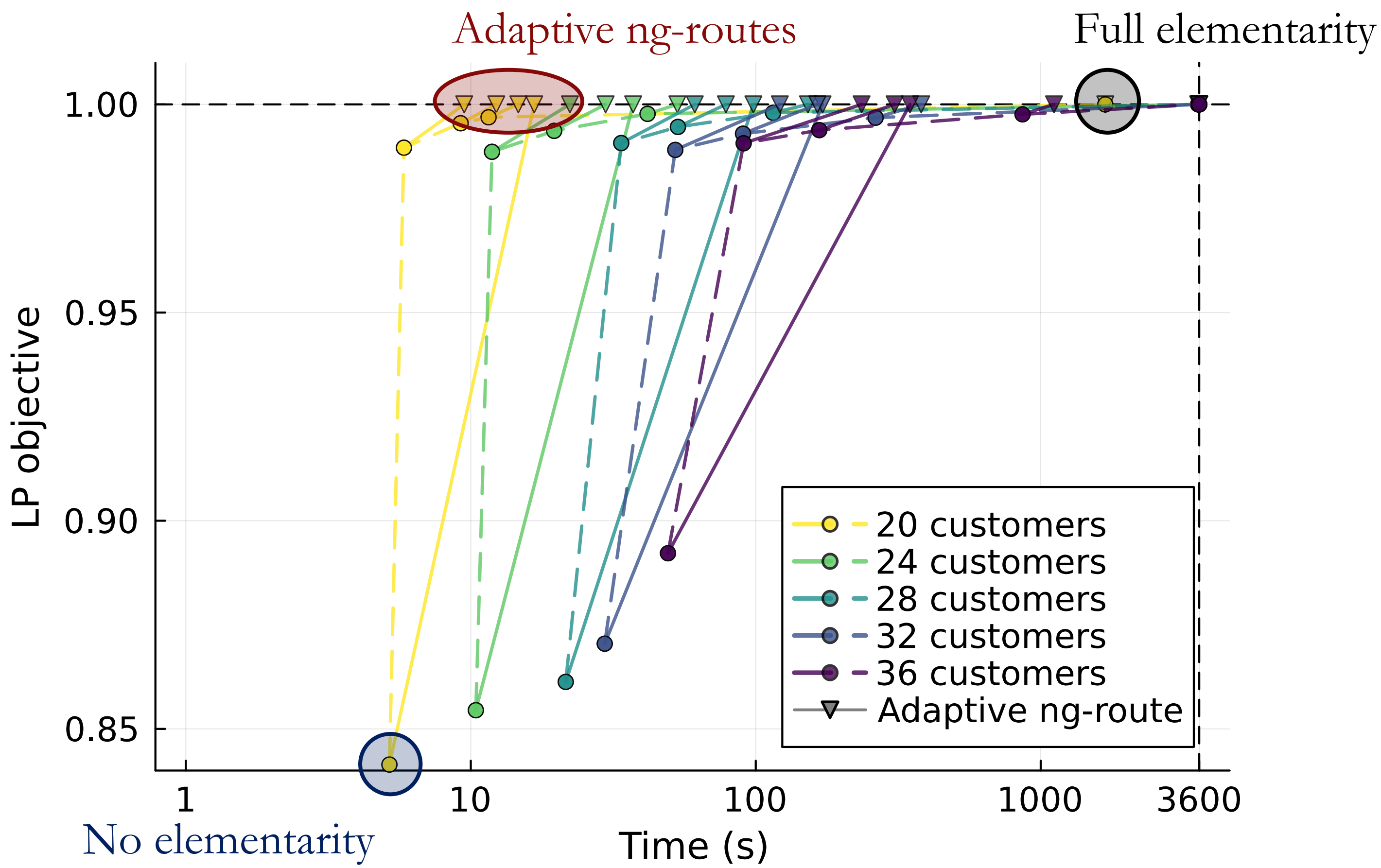
$$D(s) = \Big(\bar{c}(s),\ t(s),\ b(s),$$
$$\{\mathbb{1}\,(i \in \Pi(p))\}_i, \{\mathbb{1}\,\left(i \in \Pi^{-1}(p)\right)\}_i \Big)$$

Reason: need $D(s)$ to satisfy:

$s_1 \succcurlyeq_L s_2$ → **3** $p \oplus s_1 \succcurlyeq_L p \oplus s_2$ for any path p

[1] Baldacci, R., Mingozzi, A., & Roberti, R. (2011). New Route Relaxation and Pricing Strategies for the Vehicle Routing Problem. *Operations Research*, *59*(5), 1269–1283.

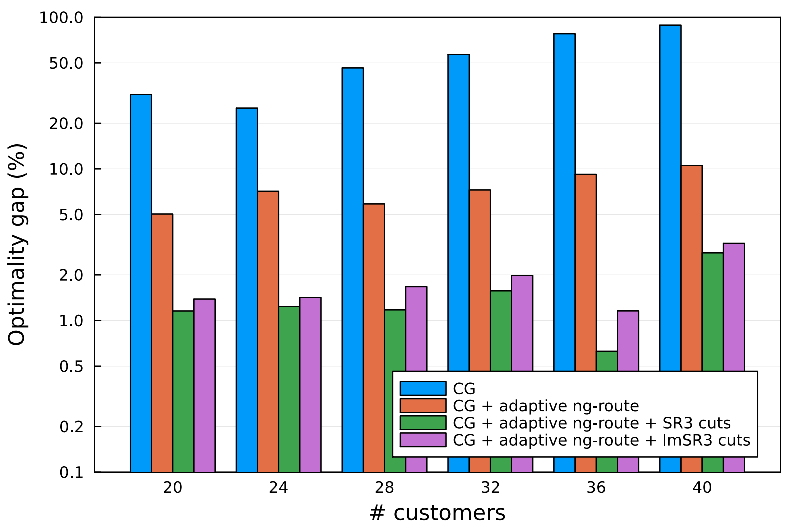
Benefits of adaptive *ng*-relaxations



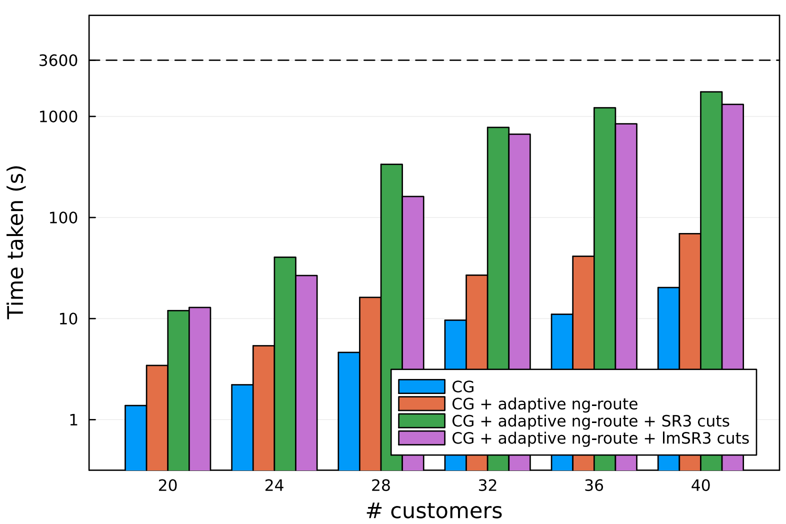
Computational results

- Weak relaxation of baseline column generation algorithm
- Strong benefits from *ng*-relaxations and cutting planes
- Scales to realistic problem instances, with dozens of nodes

Optimality gap



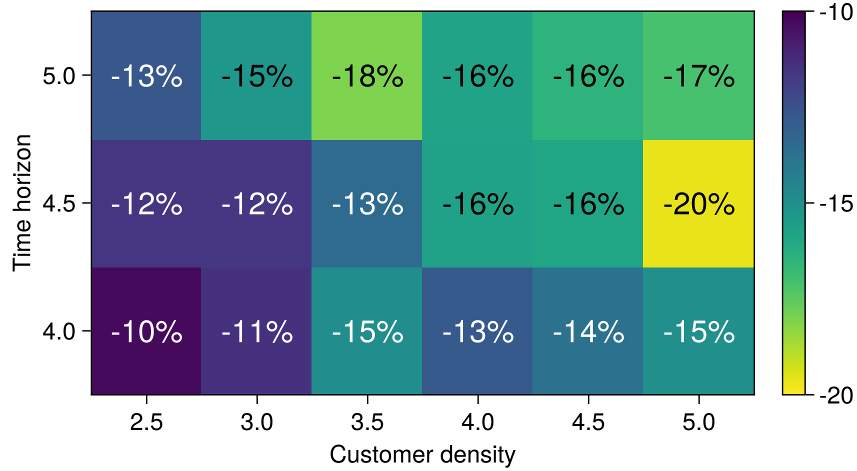
Computational times



The benefits of optimization

- Business-as-usual solution:
  - Solve a VRP w/o charge
  - Then optimize charging stations with fixed routes
- Significant improvements by **jointly** optimizing charging and routing decisions

% cost reduction vs. business as usual



Benefits from large-scale optimization algorithms to support emerging vehicle technologies and operating models toward sustainable logistics

Conclusion

Electric vehicle routing: subpath-based column generation algorithm	
Modeling	Semi-infinite formulation for electric vehicle routing: discrete routing decisions, continuous charging decisions
Optimization	• Subpath-based decomposition: two-level label-setting pricing algorithm for column generation subproblem • Forward and backward domination criteria enabling tighter relaxations: *ng*-relaxations & cutting planes • Guarantees of exactness and finite convergence
Computational results	Significantly outperforms path-based benchmark, and scales to realistic problem instances
Practical impact	Benefits of integrated routing and charging optimization

Additional slides

Subset-row cuts

- Consider a cut defined by a subset S of customers:
  - At most $\lfloor n/k \rfloor$ routes visiting at least k out of n customers[1] (Chvatal-Gomory cut of rank 1)

$$\sum_{p \in \mathcal{P}} \sum_{i \in S} \gamma_i^p z^p = |S| \Longrightarrow \sum_{p \in \mathcal{P}} \left\lfloor \frac{1}{k} \sum_{i \in S} \gamma_i^p \right\rfloor z^p \leq \left\lfloor \frac{|S|}{k} \right\rfloor$$

- *Non-robust* cuts which change subproblem structure (new duals)
  - Track resource $\sum_{i \in S} \gamma_i^p \ (mod\ k)$ for each subset S
  - When resource hits 0, subtract dual from reduced cost
  - Track $\sum_{i \in S} \gamma_i^s \ (mod\ k)$ and $\sum_{i \in S} \gamma_i^p \ (mod\ k)$ for subpaths and paths respectively:

$$D(s) = \left(\ldots, \ \{\textstyle\sum_{i \in S} \gamma_i^s\}_S \right)$$

$$D(p) = \left(\ldots, \ \{\textstyle\sum_{i \in S} \gamma_i^p\}_S \right)$$

[1] Jepsen, M., Petersen, B., Spoorendonk, S., & Pisinger, D. (2008). Subset-Row Inequalities Applied to the Vehicle-Routing Problem with Time Windows. Operations Research, 56(2), 497–511. https://doi.org/10.1287/opre.1070.0449

Limited-memory subset-row cuts

- Limited-memory subset-row cuts[1] include a *memory neighborhood* for each cut
  - Smaller state space
  - Weaker cuts
- Requires tracking forward and backward criteria

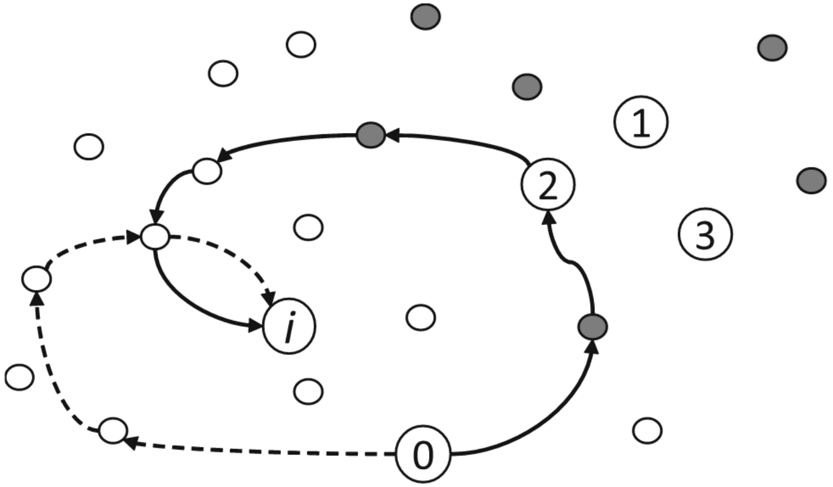


Fig. 2 Example illustrating the performance gain in the pricing when using lm-SRCs

[1] Pecin, D., Pessoa, A., Poggi, M., & Uchoa, E. (2017). Improved branch-cut-and-price for capacitated vehicle routing. *Mathematical Programming Computation*, *9*(1), 61–100. https://doi.org/10.1007/s12532-016-0108-8

Comparison with literature

Setting

[1]:

- Continuous time and charge
- Single / multiple recharges, partial / full recharging
- Time windows
- No charging costs

Our work:

- Continuous time and charge
- Multiple recharges, partial recharging
- **No** time windows
- Linear constant / heterogenous charging costs (charging τ at i costs $\delta_i \cdot \tau$)

[1] Desaulniers, G., Errico, F., Irnich, S., & Schneider, M. (2016). Exact Algorithms for Electric Vehicle-Routing Problems with Time Windows. Operations Research, 64(6), 1388–1405. https://doi.org/10.1287/opre.2016.1535

Comparison with literature

	[1]:	Our work:
Methods	• Bidirectional label-setting, with bidirectional criteria	• Unidirectional, two-level label-setting, with bidirectional criteria
	• *ng*-route relaxation	• Adaptive tightening of *ng*-route relaxations
	• 2-path cuts and subset-row cuts	• SRC and lm-SRC
	• Branching	• No branching

[1] Desaulniers, G., Errico, F., Irnich, S., & Schneider, M. (2016). Exact Algorithms for Electric Vehicle-Routing Problems with Time Windows. Operations Research, 64(6), 1388–1405. https://doi.org/10.1287/opre.2016.1535